



Trinity College

Semester Two Examination, 2017

Question/Answer booklet

MATHEMATICS SPECIALIST UNITS 1 AND 2 Section Two: Calculator-assumed

If required by your examination administrator, please
place your student identification label in this box

Student Number: In figures

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In words

Your name

Time allowed for this section

Reading time before commencing work: ten minutes

Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	99	65
Total					100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
4. Additional working space pages at the end of this Question/Answer booklet are for planning or continuing an answer. If you use these pages, indicate at the original answer, the page number it is planned/continued on and write the question number being planned/continued on the additional working space page.
5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you do not use pencil, except in diagrams.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed

65% (98 Marks)

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9

(6 marks)

- (a) The point $P(-3, 5)$ is translated by the column vectors $\begin{bmatrix} 2 \\ -7 \end{bmatrix}$ and then by $\begin{bmatrix} x \\ y \end{bmatrix}$ to $P'(9, -11)$.
Determine the values of the constants x and y . (2 marks)

- (b) Determine the single matrix that represents, in order, the composition of a rotation of 30° anti-clockwise about the origin followed by reflection in the line $y = x$. Express matrix coefficients in exact form. (4 marks)

Question 10

(6 marks)

An exam has two sections, *I* and *II*, containing 7 and 14 questions respectively.

Determine the number of different combinations of questions a candidate could choose if they must answer

(a) 2 questions from Section *I* and 3 questions from Section *II*. (2 marks)

(b) 5 questions, all chosen from the same section. (2 marks)

(c) 5 questions, with at least one question from each section. (2 marks)

Question 11

(6 marks)

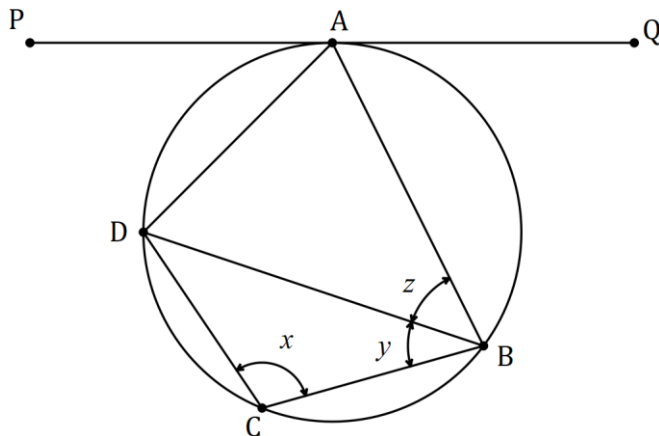
(a) A circle property says that if chords of a circle are of equal length then they subtend equal angles at the centre.

(i) Write the converse of this statement. (1 mark)

(ii) Draw a diagram to illustrate the converse statement and state whether the converse is also true. (2 marks)

(b) The diagram below shows four points A, B, C and D lying on the circumference of a circle. The line PQ is a tangent to the circle at A , $\angle BDC = 21^\circ$, $\angle PAD = 35^\circ$ and $\angle QAB = 62^\circ$.

Determine the size of angles x, y and z . (3 marks)



Question 12

(10 marks)

(a) If $\mathbf{p} = 4\mathbf{i} - 2\mathbf{j}$ and $\mathbf{q} = 3\mathbf{i} + 2\mathbf{j}$ determine

(i) the angle between the directions of \mathbf{p} and \mathbf{q} , to the nearest tenth of a degree.
(2 marks)

(ii) the vector projection of \mathbf{q} on \mathbf{p} . (3 marks)

(b) The vector $21\mathbf{i} + 5a\mathbf{j}$ has a magnitude of 29 and is perpendicular to the vector $4\mathbf{i} - 2b\mathbf{j}$.
Determine the values of the constants a and b , where $a < b$. (5 marks)

Question 13

(6 marks)

(a) Prove that $\sin 3A = 3 \sin A - 4 \sin^3 A$.

(4 marks)

(b) Hence, or otherwise, solve $3 \sin A - 4 \sin^3 A = \frac{1}{2}$, $0 \leq A \leq \frac{\pi}{3}$.

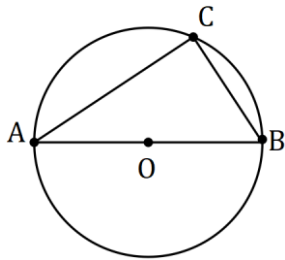
(2 marks)

Question 14

(7 marks)

- (a) Point R lies on the circumference of a circle with diameter $PQ = 51$ cm, so that $PR = 4RQ$. Determine the exact length RQ . (3 marks)

- (b) Use a vector method to prove that the angle in a semi-circle is a right-angle. (4 marks)

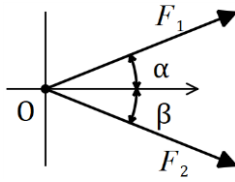


Let $\vec{OC} = \mathbf{c}$ and $\vec{OB} = \mathbf{b}$.

Question 15

(8 marks)

In the diagram below, forces F_1 and F_2 act on a body at the origin.



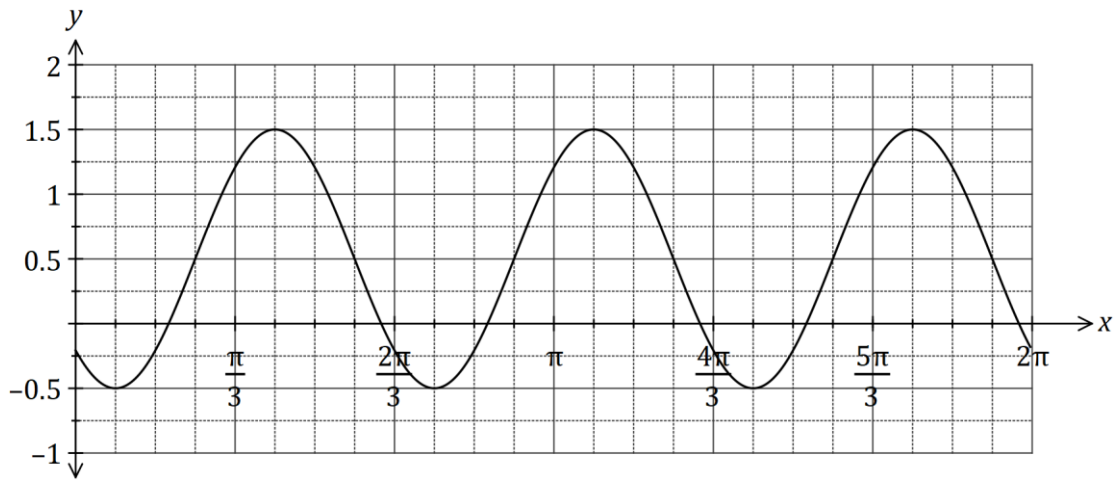
- (a) If $F_1 = 50\text{ N}$, $F_2 = 65\text{ N}$, $\alpha = 22^\circ$ and $\beta = 32^\circ$, determine the magnitude of the resultant force and the angle it makes with the positive x axis. (5 marks)

- (b) If $F_1 = 75\text{ N}$ and $F_2 = 95\text{ N}$, determine the angles α and β so that the resultant force is directed along the positive x axis and has a magnitude of 155 N . (3 marks)

Question 16

(10 marks)

(a) The graph of $y = \cos(a(x + b)) + c$ is shown below for $0 \leq x \leq 2\pi$.

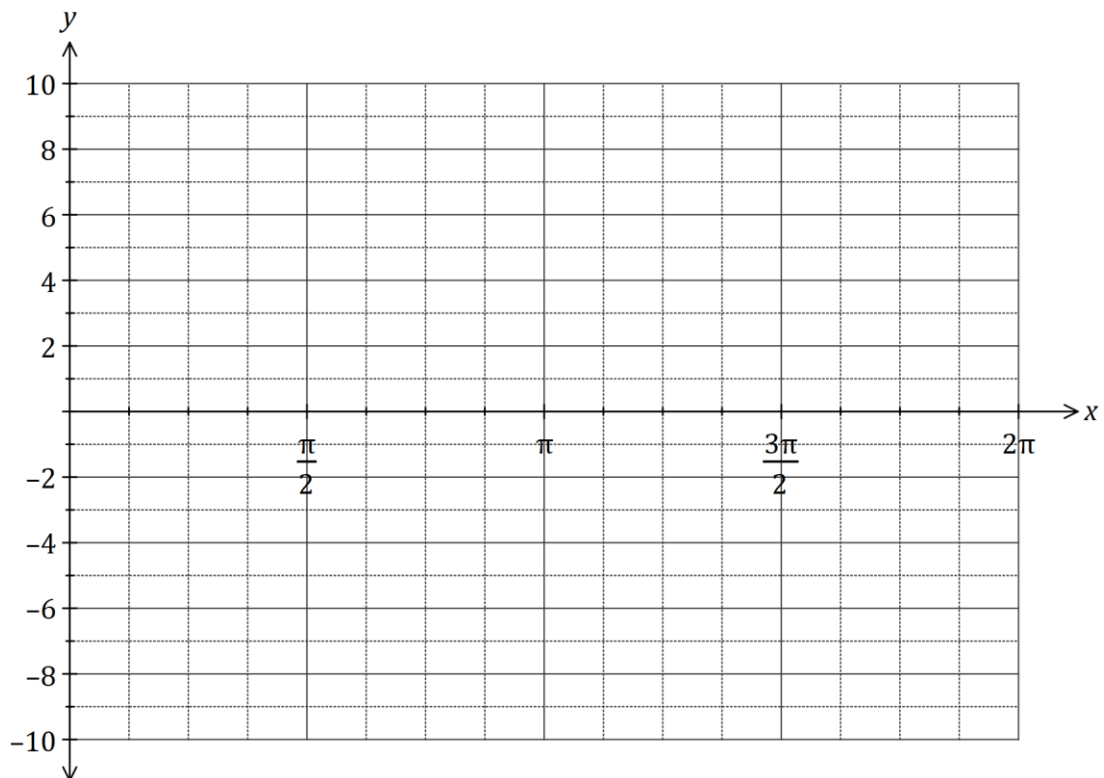


Determine the value of the positive constants a , b and c .

(3 marks)

(b) On the axes below, sketch the graph of $y = 2 \operatorname{cosec}(x - \pi)$, $0 \leq x \leq 2\pi$.

(3 marks)



- (c) The displacement, x cm, of a particle from a fixed point O varies with time, t seconds, according to the model $x = 4 \sin(3\pi t) - 7 \cos(3\pi t)$, $t \geq 0$. Determine
- (i) the initial displacement of the particle from O . (1 mark)
- (ii) the exact amplitude of the motion. (1 mark)
- (iii) the period of motion. (1 mark)
- (iv) the first time that the particle passes through O . (1 mark)

Question 17

(9 marks)

Triangle ABC has vertices $A(1, 2)$, $B(4, -1)$ and $C(5, 3)$.

- (a) The vertices ABC are transformed to $A'B'C'$ using matrix $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$. Write down the new coordinates of the vertices and describe the transformation. (4 marks)

- (b) The vertices ABC are transformed to $A''B''C''$ by matrix M so that the new coordinates of the vertices are $A''(-4, 3)$, $B''(2, 12)$ and $C''(-6, 15)$.

- (i) Determine the transformation matrix M . (3 marks)

- (ii) If the area of triangle ABC is k square units, express the area of triangle $A''B''C''$ in terms of k . (2 marks)

Question 18

(7 marks)

- (a) How many numbers must be chosen from the set of integers between 1 and 2017 inclusive to be certain that one of the numbers chosen is a multiple of 10. (3 marks)

- (b) A number is formed using four different digits chosen from those in the number 23 814. Determine how many different numbers can be formed that are

(i) even. (1 mark)

(ii) greater than 8 000. (1 mark)

(iii) even or greater than 8 000. (2 marks)

Question 19

(10 marks)

- (a) Trapezium $OPQR$ has parallel sides PQ and OR . M is the midpoint of OQ and N lies on QR so that $RN:NQ = 1:2$.

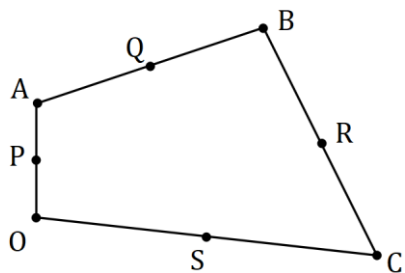
Given that $\overrightarrow{OP} = \mathbf{p}$, $\overrightarrow{OR} = \mathbf{r}$ and $\overrightarrow{PQ} = 2\mathbf{r}$, determine the following in terms of \mathbf{p} and \mathbf{r} .

- (i) \overrightarrow{OM} . (2 marks)

- (ii) \overrightarrow{ON} . (2 marks)

- (iii) \overrightarrow{NM} . (2 marks)

- (b) Quadrilateral $OABC$ is shown below, where P , Q , R and S are the midpoints of the sides OA , AB , BC and OC respectively. Let $\vec{OP} = \mathbf{a}$, $\vec{AQ} = \mathbf{b}$ and $\vec{OS} = \mathbf{c}$.



Show that $\vec{PS} = \vec{QR}$.

(4 marks)

Question 20

(8 marks)

The sum of the first n terms of the sequence $2 + 8 + 14 + 20 + \dots + (6n - 4)$ is $n(3n - 1)$.

(a) Show that this statement is true when $n = 5$. (2 marks)

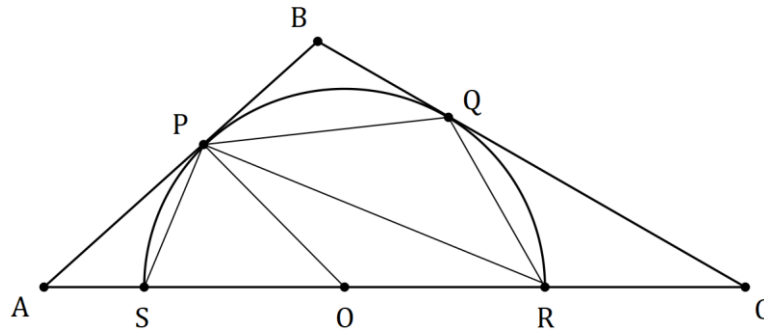
(b) Use mathematical induction to prove the statement is true for $n \in \mathbb{Z}, n \geq 5$. (6 marks)

Question 21

(6 marks)

The diagram shows a semi-circle, with diameter SR and centre O , circumscribed by triangle ABC , in which $\angle BAC = 42^\circ$ and $\angle BCA = 32^\circ$.

Determine, with reasons, the size of angles $\angle PRO$ and $\angle PRQ$.



Additional working space

Question number: _____

Additional working space

Question number: _____

